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Two tides a day

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Doe, Nick, Winter tides, *SHALE* 10, pp.33–36, January 2005.

Doe, Nick, Summer tides, *SHALE* 5, pp.45–7, December 2002.

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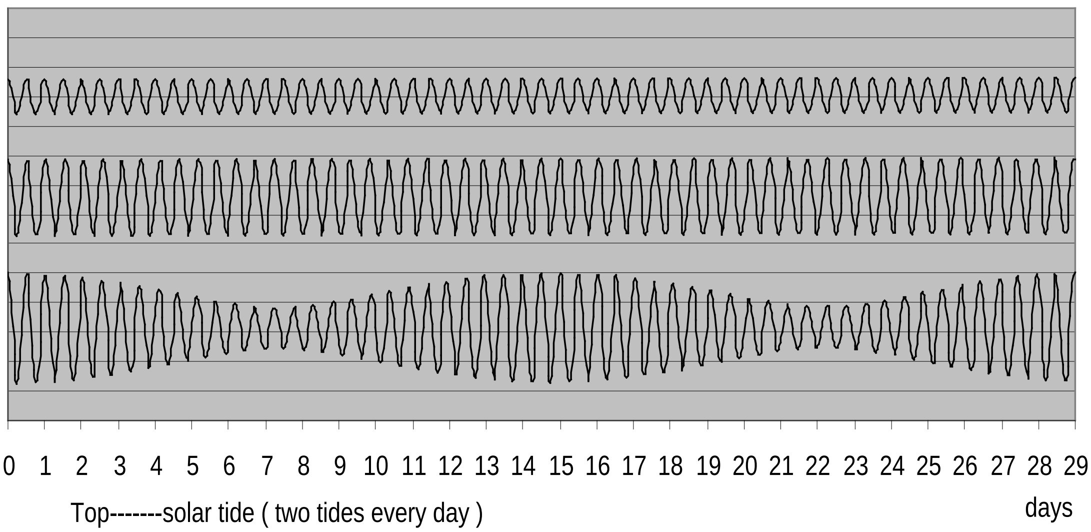
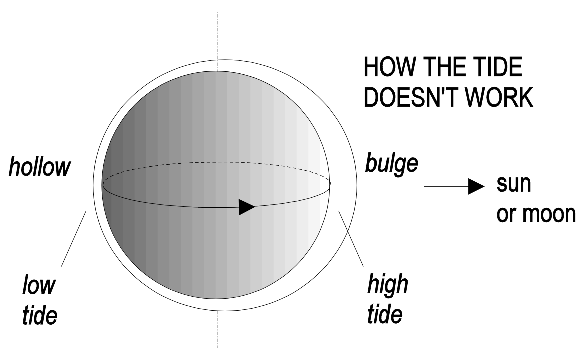
Two tides a day?

by Nick Doe



Top-------solar tide (two tides every day)
Middle---lunar tide (two tides every 24 hrs. 50 mins.)
Bottom---solar and lunar tides added together
(on average, two tides every 24 hrs. 50 mins.,with *spring* and *neap* tides)

Why are there two tides—or tidal cycles—a day and not just one? This isn't the easiest of questions to answer, but here's my best shot at it. Let's start by looking at the (false) one-tide-a-day theory.



Here we have the earth and its ocean with the sun, or moon, away to the right. The gravitational pull of the sun, or moon, attracts the water creating a bulge on the right, and a hollow on the left. To the observer on the earth, the bulge is a high tide, and the hollow is a low tide. Once a day, the observer passes by these places as the earth spins around on its polar axis. So there's only one tidal cycle a day…right?

Well no, there are of course two. So what's wrong with this theory? What's wrong is that it supposes that the earth is firmly "nailed" to a particular point in space. According to the theory, it's OK for the water to move in response to the gravitational pull of the sun or moon, but the solid earth ignores it. But that's just not how it happens. We have to consider how the whole earth moves in response to the gravitational pull of the sun or moon, not just the water.

Step one

Tides are not caused by anything on earth. The gravitational pulls of the sun and moon cause tides, the reason being that the directions and magnitudes of these gravitational pulls change as the sun and moon wander about the sky.[1] The gravitational pull of say, Mount Benson, may be stronger locally than the pull of any extra-terrestrial object, but unlike extra-terrestrial objects, Mount Benson doesn't move about very much, and so doesn't cause *changes* in the level of the sea.[2]

The second thing to remember is that it's only *differences* in the gravitational pulls of extra-terrestrial objects (the sun and moon) at different points on the earth that matter. If the pull is the same the world over, the water has no incentive to go anywhere in particular, and so there isn't a tide.

The gravitational pull of the sun or moon at any point depends on how far from the sun or moon the point is. For instance, when the sun is directly overhead (in the tropics), you are nearer to the sun than is the centre of the earth, which is directly below your feet. The average attraction of the sun on a one-kilogram weight in this situation is 605.748 milligrams. At midnight in the same place, you are now further from the sun than is the centre of the earth. The average attraction of the sun on the one-kilogram weight is then only 605.645 milligrams. The difference between day- and night-time pulls from the sun is 0.103 milligrams. Not a lot, but then, not zero.

Similarly, when the moon is directly over your head (either in or not very far from the tropics), you are nearer to the moon than is the centre of the earth. The average attraction of the moon on a one-kilogram weight in this position is equivalent to 3.499 milligrams. Twelve and a half hours later, when the moon is on the opposite side of the earth, the average attraction drops to 3.274 milligrams, a difference of 0.225 milligrams.

Notice that in spite of its greater attraction, the difference for the sun (0.103 mg) is only 46% of the difference for the moon (0.225 mg)—the sun is much further away than the moon, so changes in position are less significant for the sun than they are for the nearby moon. This is why, in general, the tide due to the sun is only 46% as strong as that due to the moon. You can see this in the graphs at the start of this article, which show idealized tides.

Step two

There are two ways of looking at the gravitational forces acting on the earth and the way the earth moves in response. Because there are two ways, there is often confusion and unnecessary disagreement. So let's sort this out.

The stationary observer's perspective

Think about a spacecraft orbiting the earth with its rocket engines turned off. You might say that the only force acting on the spacecraft is the "real" force of gravity. If you say this, then you also say that the spacecraft is accelerating, that is, it is constantly changing direction as it moves in a circle (or ellipse) as a consequence of this real force. This is the perspective of someone out in space watching the

[1] The planets cause tides too of course, but the strongest planetary tide, that of Venus, is some 19000 times weaker than that of the moon. On Gabriola, the largest Venusian tide, is about the same as the thickness of a piece of paper (0.005")—not exactly something that's going to cause widespread flooding.

[2] If you reckon Mount Benson to be a cubic mile of rock, you would have to get within four kilometres of it before its pull rivalled that of the moon, so it's not a good example, but you get the general idea (I hope).

spacecraft go round and round. It's the classic Newtonian perspective.

The moving observer perspective

But you can also say that there are really two forces acting on the spacecraft. One is the force of gravity—the *centripetal* or inward force—and the other is the *centrifugal* or outward force that is tending to make the spacecraft fly off in a straight line. If you say this, you say that the net force is zero, the "real" force of gravity being balanced by the equal and opposite "fictitious" centrifugal force.[3] This is the perspective of someone inside the spacecraft who feels that he or she is floating freely in zero-g and who, if without a window, would be unaware that the spacecraft is moving.

In spite of the "zero-g" label, the force of gravity is *not* zero in the spacecraft; it just appears to be zero because the frame of reference—the spacecraft—is moving freely in response to this force.

Both accounts describe exactly what is happening, but in different ways and from different perspectives.

Wanna lose weight?

Weight is the gravitational attraction of the earth, less whatever it takes to stop you whirling off into space as a result of the earth's spin. The closer you are to an object, the greater its gravitational attraction—this is Newton's famous inverse square law.

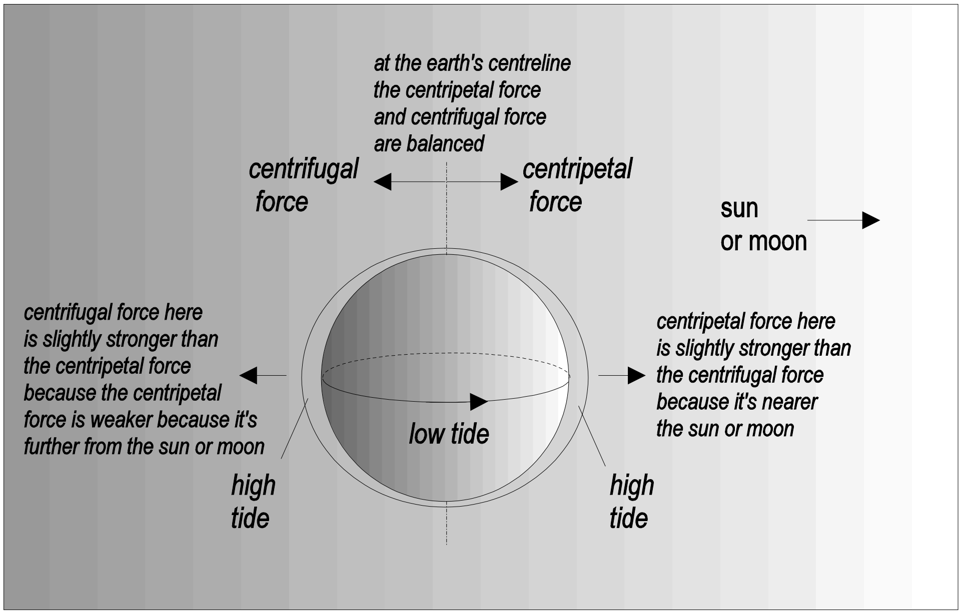
Let's suppose that, at sea level on Gabriola, you weighed exactly 65 kilograms (about 140 lb.). Then a quick way to lose weight would be to go to the top of the island. There, you'd weigh three grams less (the weight of two dimes) because up there you're 160 metres further away from the centre of the earth than you are by the sea.

A better weight-losing idea than hanging out on Chernoff is to fly down to the equator. The earth is not a perfect sphere—it has a bulge at the equator—so you are 21.5 kilometres further away from the earth's centre at the equator than you are at the north pole. Your Gabriolan weight of 65 kilograms would drop by a handsome 250 grams (about half a pound) because of this. And there's a bonus. Because the spin of the earth is greater at the equator than at the latitude of Gabriola, you'd lose an additional 80 grams as the earth tried to throw you off into space, for a total saving of 330 grams. That's the weight of a small box of chocolates! People are, quite literally, light-headed when they go south on vacation.

Just for comparison, the average attraction of the *sun* on a 65-kilogram body is about 40 grams—the weight of a roll of wine gums. However, you don't benefit from this when the sun's overhead because, as far as the sun is concerned, you're in orbit and in "zero-s". The average attraction of the *moon* on a 65-kilogram body is 220 milligrams, but again you don't lose this weight because you're in "zero-l", the lunar version of "zero-g".

The best that any of the planets can muster is 2 milligrams (about 36 grains of sand). That's the attraction of *Jupiter* to your body when the planet is closest to the earth.

[3] Scientists sometimes use words like "real", "imaginary", and "fictitious" as distinguishing labels without necessarily implying that these terms are to be understood literally. BC Hydro's transmission lines for example have "imaginary" (reactive or non-power carrying) currents flowing through them, but the utility nevertheless has to spend "real" money to deal with them. It is said that a "real" force is one that is associated with matter, and doesn't vanish when the frame of reference is not accelerating (inertial), but Einstein more-or-less put paid to that idea when he postulated that the "real" force of gravity is indistinguishable, even in principle, from motion in a noninertial frame of reference. Modern string theory may yet reveal similar geometric equivalencies for the other nongravitational "real" forces.



Step three—moving observer

Let's now take another look at the little diagram on page 25, bottom left, and compare it with the diagram above.

Here we are showing the "moving observer perspective" of what's happening. The earth as a whole is experiencing, not zero-g, but "zero-s" (no solar gravity), or if we're thinking about the moon, "zero-l" (no lunar gravity). Again, this doesn't mean that there is no solar or lunar gravity, just that the frame of reference, the moving earth, is responding freely to these forces and so they appear not to exist.

At points on the earth that are the same distance from the sun or moon as the centre of mass of the earth, there is a perfect balance between the gravitational pull (the centripetal force) and the centrifugal force that is tending to make the earth fly off into space in a straight line rather than being in orbit.

Because the earth moves as a whole from place to place in space, the outward centrifugal force is the same everywhere on earth, but at points nearer the sun or moon than the earth's centre, the inward gravitational pull of the sun or moon is a little stronger. Similarly, at points further away from the sun or moon than the earth's centre, the gravitational force of the sun or moon is, as we have seen, a little weaker.

What's the result of these small imbalances? On the right of the diagram, the gravitational pull of the sun or moon is a little stronger than the centrifugal force, so the water is pulled away from the earth creating a bulge. On the left, the gravitational pull is a little weaker than the centrifugal force, so the water tends to fly off into space away from the earth creating a second bulge. Hence, two bulges and two tides a day.

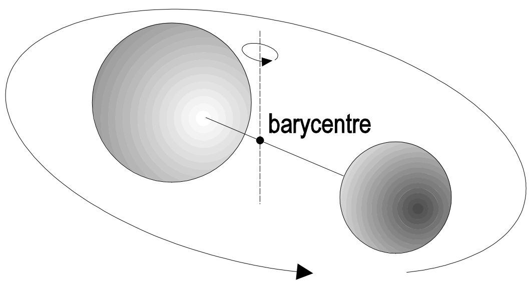
Step four—loose ends

Many people will be happy with that explanation, but for those who don't believe in "centrifugal" forces, let's try it again. But be warned, the explanation is a little harder to follow, even though the end result is exactly the same.

We start by taking a pencil and crossing out the (leftward) centrifugal force in the diagram leaving us with the centripetal force, which points towards the sun or moon. Now in the case of the sun, it's fairly straightforward to identify what this force is. The gravitational pull of the sun is what keeps the earth in orbit around the sun, just as the gravitational pull of the earth keeps a spacecraft in orbit around the earth. But if we substitute the moon for the sun, what then? It's the moon that orbits the earth—not the other way round. So what is this centripetal force doing when the object to the right is the moon? It's time to talk about "barycentres".

Barycentres

The earth orbits the sun—the moon orbits the earth. That's almost right, but not quite.



When two objects that gravitationally attract each other, as do the earth and sun, and the earth and moon, the objects orbit around their *common centre of mass*, which is known technically as the *barycentre* of the system.

Think of two dumbbells connected by a short rod. Then, if the balls are equal in mass, the barycentre is in the centre of the rod that joins them—it's the pivot point around which the dumbbells would orbit if they were in space. If one ball is significantly bigger than the other, then the pivot point moves toward the more massive ball.

The dumbbell analogy is not perfect, because it supposes that the balls always show the same face to the barycentre. This happens to be true in the case of the moon, but not so in the case of the earth and sun, both of which rotate on their polar axes independently of the barycentric rotation.

For the earth-sun system, the barycentre around which both rotate is actually the *barycentre of the solar system*. Because the sun is so massive, the barycentre of the solar system lies pretty close to the centre of the sun, but not exactly there.[4]

For the earth-moon system, the barycentre around which both rotate lies pretty close to the centre of the earth, but not exactly there.[5]

The important point is that in *both* cases, sun and moon, the earth is orbiting a barycentre. This orbiting is obvious in the case of the sun, but not so in the case of the moon. We tend to think of the moon's orbiting the earth as having no effect on the position of the earth in space, but in reality, each one of us is doing a little circular motion, 4667 km in radius (the width of Canada), once a month[6] in response to the gravitational pull of the moon. Unless we are astronomers, we don't notice this motion because it is obscured by the spin of the earth.

Don't be confused, by the way, into thinking that because the centre of mass of the earth orbits the barycentre, every point on earth orbits the barycentre. As you sit in your armchair, your centre of barycentric rotation is offset from the barycentre in keeping with your own offset from the centre of the earth. Your personal centre of barycentric rotation always lies, as close as makes no difference, directly between you and the sun or moon.

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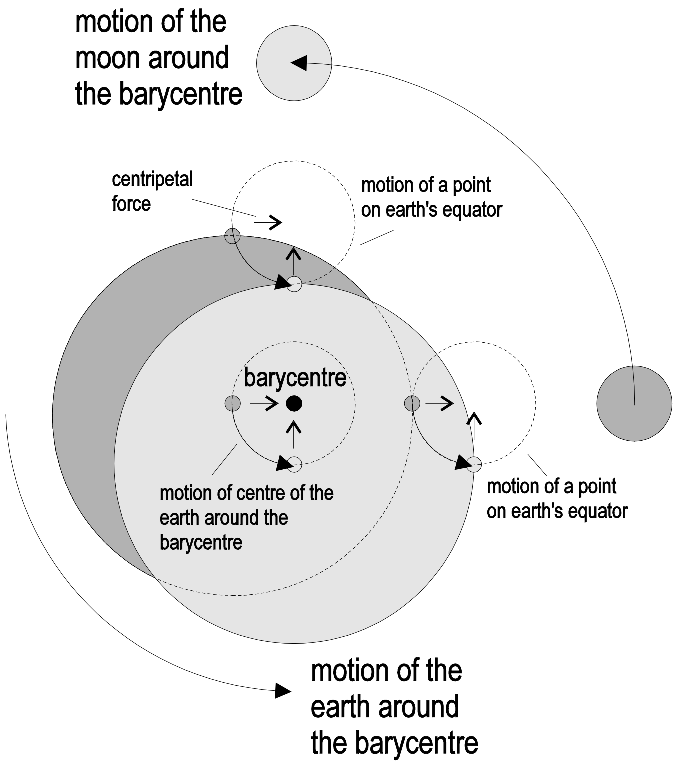
4 The movement of the sun in its tight orbit around the barycentre of the solar system—once every 180 years or so—is what very distant astronomers would use to infer the presence of the planets, even if they couldn't see them.

5 Even if the moon were invisible, a distant observer would know that the moon was there by watching the earth move once a month in a tight orbit around the earth-moon barycentre in addition to moving around the sun.

6 Technically, a sidereal month of 27 days, which is how long the moon takes to go around relative to the stars.



The diagram shows the rotation of the earth and moon around their common *barycentre* as seen from above the earth's north pole. Two positions in the rotation are shown starting with images in the darker grey, and finishing with images in the lighter grey. The positions are a quarter of an orbit (90 degrees or one week) apart. For clarity, the once-a-day spin of the earth is not shown.

Because the earth is more massive than the moon, we tend to think of the moon orbiting an earth that is fixed in place, but in fact both earth and moon make a once-a-month orbit of the barycentre. The earth's motion is like that of a wheel with an axle that is slightly offset from the centre of the wheel. If you stop the spin of the earth around its own axis, which is something quite separate, and follow the motion of a point on the earth's equator, you can see that each point also does a little circular orbit once a month. This circular motion requires an inward (centripetal) force and this is provided by the gravitational attraction of the moon. The centripetal force always acts both in the direction of the moon, and toward the centre of the circular motion.

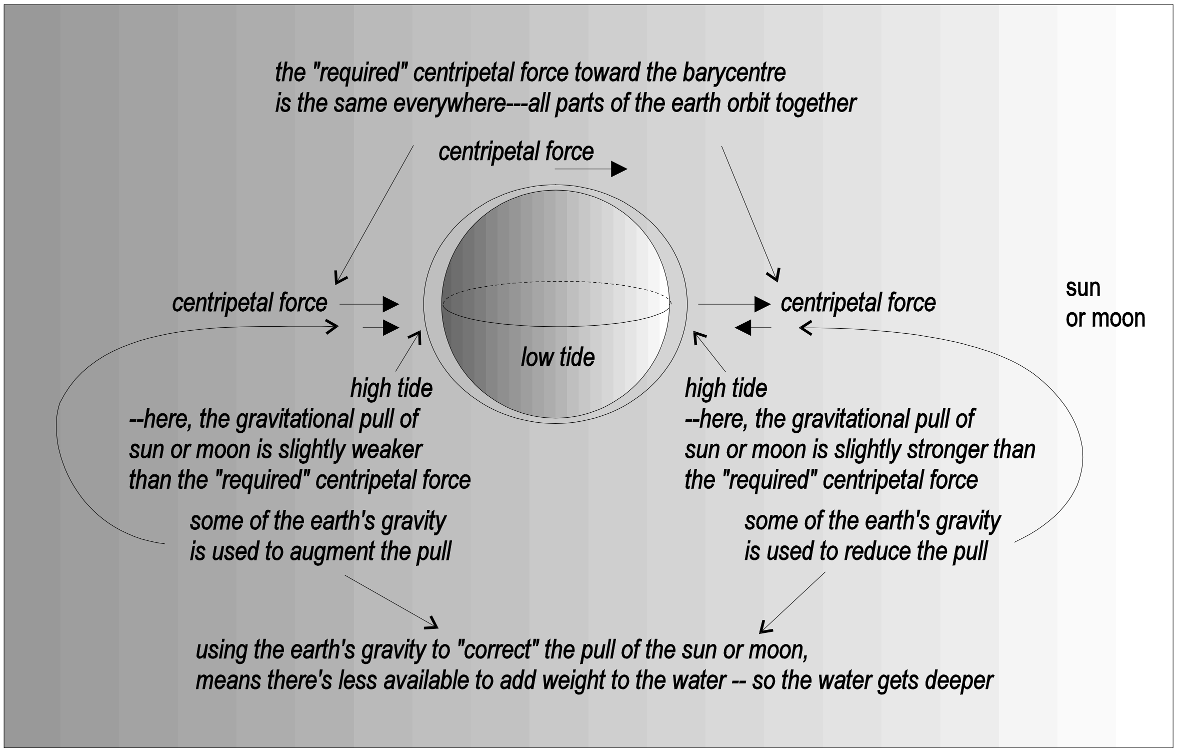
Each of the little circular motions requires an inward (centripetal force), and that is what the moon's gravitational pull provides.

The tidal forces

Now let's look at the centrifugal-force-free diagram on the next page. Because the lengths of the orbital paths around a barycentre, and the time required to complete an orbit around a barycentre, is the same for all places on earth, the inward (centripetal) force required is the same everywhere. If it weren't, the earth would be pulled apart. But the centripetal force is supplied by the gravitational pull of the sun and moon, and this differs slightly from place to place.

On the side of the earth nearest the sun or moon (on the right of the diagram), there is a slight excess of pull. Fortunately, this does not result in water streaming off into space because some of the earth's gravity *counteracts* this excess pull, but it does result in the water weighing a bit less. The pressure at the bottom of the ocean is thereby reduced, and water is encouraged to flow to that part of the earth and create a high tide.

On the side of the earth furthest from the sun or moon (on the left of the diagram), there is a slight deficiency in the pull. Again, as you may have noticed, this does not result in water streaming off into space, and it doesn't because, this time, some of the earth's gravity *augments* the pull of the sun or moon. As on the other side of the earth, this means there is slightly less of the earth's



gravity available to attract the water, so it consequently weighs a bit less, which results in a lower pressure at the bottom of the ocean, which encourages water to flow to that part of the earth and thereby create the second high tide.

If you look at the mathematics of this, you will see that this explanation is no different from the earlier one involving centrifugal forces—it's up to you which you prefer.

Tidal ranges

We can, if you like, have a stab at calculating how high, in theory, the high tides are. If we reckon that, compared to the centre of the earth, the increase in gravitational attraction of the sun and moon combined (assume a new moon) is 0.164 kg per tonne[7] at the point nearest the sun and moon; and the density of the sea is roughly one tonne per cubic metre; and the average depth of the world's oceans is about four kilometres; then the theoretical height of the bulge is 0.67 metres (2.2 feet) above mean sea level.

Real tides are much bigger than that of course, and that's because real tides are the accumulated result of several theoretical tides. Just as a bunch of kids in a swimming pool can get the water to slosh from one end to the other by timing their back and forth movements, so the sun and moon build up their respective tides by their rhythmic actions. If you started with a perfectly still ocean, it would take the best part of a week before the tide built up to the levels we actually see. And of course, this happens in different ways in different places depending on the topography of the local shoreline, the bathometry of the ocean, and the intricacies of the sun and moon's movements in the sky. But that's a topic for another day. ◊

[7] =0.5*(103+225) mg per kilogram