Context:
Captain Cook, longitude, Nautical Almanac errors, 1778

Citation:
Doe N.A., *An Analysis of Captain Cook's Longitude Determinations at Nootka, April 1778*, LIGHTHOUSE, Journal of the Canadian Hydrographic Association, 48, pp.21-29, Fall 1993.

Copyright restrictions:

Errors and omissions:
The original printed version had some layout problems and so the original page numbers are not preserved in this copy.

Later references:

Date posted:
April 1, 2011.

Author:
Nick Doe, 1787 El Verano Drive, Gabriola, BC, Canada V0R 1X6
Phone: 250-247-7858, FAX: 250-247-7859
E-mail: nickdoe@island.net

AN ANALYSIS OF CAPTAIN COOK'S LONGITUDE DETERMINATIONS AT NOOTKA, APRIL 1778

Nicholas A. Doe

Summary:

The British naval officer Captain James Cook is justifiably famous for his contributions to the art of navigation in the 18th century. However, the method that he helped develop to measure longitude, which was based on observations of the Moon, required a great deal of calculation in contrast to the chronometer method that was also being developed at the same time. Although some attention has been given to the role of instrument errors in determining the accuracy of Cook's longitude determinations, the role of computational errors has been neglected. The lunar distance method was so computationally-intensive that errors were inevitable. This paper examines the results of a re-working of Cook's calculations which brings to light a number of interesting mistakes and mis-recordings of observational data and shows that, by modern standards, "paperwork" mistakes were quite common. The paper also identifies and corrects the principal source of the error in Cook's determination of the longitude of Nootka.

AN ANALYSIS OF CAPTAIN COOK'S LONGITUDE DETERMINATIONS AT NOOTKA, APRIL 1778

By Nicholas A. Doe

The sheets were all read until no error could be found; therefore I hope very few have escaped; but it is highly probable there will be some among such a multiplicity of figures.
W.Bayly, Commissioners of Longitude's astronomer on Cook's third voyage.

...that gentleman's (Bayly's) *book is full of errors...*
Lieutenant King, chief astronomer and later captain on Cook's third voyage.

...W.W. has seen many bad reckonings, but few so bad as it (King's log) *contains.*
W.Wales, astronomer on Cook's second voyage.

The aboriginal people of the west coast of Vancouver Island have known for at least 4000 years where their village of Yuquot is. But for Captain James Cook and his crew, who arrived there in April 1778, the location of King George's Sound, or Nootka as it became to be known, had to be determined by its latitude and longitude. For the 18th-century British explorers, Nootka was thousands of kilometres from anywhere.

Living now as we do in an age when the art of haven-finding demands little more of us than the ability to push buttons and read displays it is, perhaps, difficult for us to look back to a time when the development of techniques for measuring geographic location was cause for excitement. But when Cook embarked on the first of his famous voyages to the Pacific in 1768, European explorers and the interested public alike, were exuberant that it was at last possible to sail across thousands of kilometres of open ocean to never-before-visited lands and islands, guided only by two simple numbers.

The latitude of any place is its angular distance north or south of the Earth's equator. Its measurement is relatively easy, for the further north you are, the proportionately lower will the Sun and stars in the sky to your south appear. Longitude however, is far more difficult to determine accurately. Longitude is the angular distance east or west of a given line (meridian), running from the north to south pole. Nowadays the universally adopted prime meridian is the one through the observatory at Greenwich in England, but before this was agreed upon in 1884, navigators selected their own prime meridians. Eighteenth-century Spanish charts of the coast of British Columbia for example, have longitudes marked relative to the meridians of Cadiz, Tenerife, San Blas, and occasionally Paris.

The measurement of longitude amounts essentially to measuring the difference between local time and Greenwich time. The later that local noon occurs after noon at Greenwich, the further west you are. Hence to measure longitude, one could set one's clock to be twelve at the moment the Sun is due south, and then phone a friend at Greenwich who had done the same, and compare times. Alternatively, you could fly to England, set your watch by the Sun, bring it back home, and check it at noon. In the city of Vancouver, if we neglect small variations in the timing of noon due to the Earth's slightly elliptical orbit, your watch would be 8^h 12^m 24^s fast, which corresponds to a longitude of 123°06'W. [Note: 1° (degree) = 60' (minutes of arc) = 4 minutes of time].

Captain Cook had neither telephones nor airlines, nor quartz digital watches that could keep good time for the many months it took to reach the Pacific coast of North America. He did have good time-keepers, chronometers as they are called, and these he used extensively for measuring the relative longitudes of places not too far apart, but eventually cumulative errors made it necessary to re-calibrate them. For the time signals he needed to do this, he had to look to the sky.

The method that the British Navy used to tell the time at Greenwich is known as the method of lunar distances. Basically this involves measuring the position of the Moon in its monthly orbit around the Earth, and then using pre-calculated tables to determine the predicted time for the Moon to be at that position. The Moon was most often located by measuring the angular distance between it and the Sun, but its distance from selected stars was also used.

Other methods for finding Greenwich time, such as observing the timing of eclipses of the moons of Jupiter, or the timing of the occultation of stars by the Moon, were more accurate, and were used successfully on the west coast by the Spanish. However, these methods could not be used at sea because of the difficulty of aiming a high-powered telescope from a swaying deck. The instrument used for measuring angular distance, the sextant, overcomes this problem by using a system of mirrors that brings the images of the two bodies being observed together, irrespective of the unsteadiness of the hands that are holding it.

The Moon completes its orbit around the Earth, on average once every 27.5 days relative to the (fixed) stars, and once every 29.5 days relative to the Sun. These times are different because as the Earth moves around the Sun in its annual orbit, the direction of the Sun, relative to the background of stars, slowly changes. The movement of the Moon is most familiar to us as the gradual progression from new moon, when Sun and Moon appear in the same direction and set together in the evening, to full moon, when they are in opposite directions and the Moon is high in the sky throughout the night. The transition from full moon back to new moon is less conspicuous because, during this part of its orbit, the Moon is only visible to the casual observer in the early morning hours.

Compared to the Sun's 24 hours, it takes the Moon, on average, 24 hours and 50 minutes to reappear in the same position in the sky each day. Consequently, although the Moon rises in the east and sets in the west, just like the Sun, it does so more slowly, and it always appears further to the left of where it was 24 hours earlier. It is this constantly changing position, relative to the Sun and stars, that the 18th-century navigators sought to measure, and thereby determine Greenwich time, and hence their longitude.

Captain Cook spent four weeks at Ships' Cove (now Resolution Cove) on Bligh Island in Nootka Sound (49°36.4'N, 126°31.7'W). There, he repaired and re-provisioned his ships, met and traded with the native people, and did all the things that famous explorers do. During this time, he and his chief astronomer, Second Lieutenant James King, made 91 sets of measurements of their longitude. William Bayly, the astronomer aboard HMS *Discovery*, the ship accompanying HMS *Resolution*, made a further 31 sets of measurements. Since each set usually involved the averaged value of six observations of the position of the Moon, the sum total of observations made could well have been in excess of 600.

The translation of a measurement of the Moon's position to a determination of longitude was no simple matter. This is because the Moon's apparent location is modified by both refraction and parallax.

Although the distorting effects of refraction in the Earth's atmosphere are much less than those of say, water or glass, we nevertheless see the Sun, Moon, and stars as though through a giant lens; and the navigator must correct for this in the course of making his calculations.

Parallax was an especially difficult complication to the method of lunar distances. Because the Moon is so close to us relative to the other heavenly bodies, the apparent direction of the Moon changes as we move about on the Earth's surface. In order to be able to use a universally applicable set of tables describing the lunar orbit, the navigator was obliged to calculate from his observations, the direction in which the Moon would appear, if it were to be observed from a position corresponding to the centre of the Earth.

The step-by-step instructions for computing corrections for refraction and parallax occupied 17 pages in one early navigators' manual, and resembled, in their complexity and obscurity, a modern income tax form. If we remember that this work had to be done for each of the 122 sets of measurements made at Nootka, in cramped and poorly lit quarters, without the aid of calculators or computers, we can get some idea of the great investment Cook made in fixing his position accurately.

Yet for all that, Cook's determination was not perfect. His journal records the longitude of Ships' Cove as 126°42.5'W (233°17'30.5"E in the old notation), which is 10.8' (12.9 km) west of its true position. Moreover, when Cook's midshipman, George Vancouver, returned to the coast 14 years later and re-determined Nootka's longitude over a hundred times (636 observations) using the same techniques, his result was 8.6' (10.3 km) too far east. This compares poorly with the Spanish determinations which were correct to one or two minutes of arc.

The source of the difficulty of the method of lunar distances is the slowness of the Moon's motion. It takes the Earth only four seconds to rotate through one minute of arc, but the Moon takes two minutes to move the same amount relative to the stars and Sun. This is a ratio of 30:1; hence, in order to achieve a longitude accuracy of one minute of arc (one sixtieth of a degree), the Moon's position has to be pin-pointed with an accuracy 30 times better than this, i.e about two seconds of arc. Even modern sextants are about five times less accurate than this, and the uncertainties in the refraction corrections preclude any further improvement.

By taking hundreds of observations and averaging the results, the 18th-century navigators were attempting to reduce the effect of the random errors of their instruments. In principle, this strategy was a good one. The random error of the average of 625 readings will, on average, be less than the error of the individual readings by a factor of $\sqrt{625} = 25$. So, given a basic sextant accuracy of one minute of arc, it should have been possible by averaging over 600 results to have improved the accuracy sufficiently to make a longitude reckoning of the same order of accuracy.

So where's the snag? Surveyors and hydrographers of course hardly need to be told. The snag is that the errors must be truly random for this technique to work. If every result contains exactly the same error, then so will the average, and this will be so, no matter how many results are averaged. Captain Cook's determinations show every sign of containing such a non-random (systematic) component, which averaging could, and did not eliminate. The author's task in analysing Cook's results was to identify precisely this systematic error.

The source material for this work is contained in Bayly's book, *The Original Astronomical Observations...*, published in 1782 by the Commissioners of Longitude. It contains 350 pages of details of astronomical, horological, meterological, oceanographic, geomagnetic, and geodetic observations made during the course of the third voyage. Each of the 122 sets of lunar distance observations made at Nootka is summarized in a 14 column entry; these are the date, time according to the deck watch, apparent time (i.e. true local solar time), the lunar distance from Sun or star, the altitude of the Sun or star, the altitude of the Moon, the sextant used, the sextant index error, the barometric pressure, the temperature, the identity of the observer (Cook, King or Bayly), the latitude of Nootka (which was accurate to 0.3'), the deduced longitude, and the identity of the star, if not the Sun, from which the lunar distance was measured. Although the original editors of Cook's journal make no mention of Bayly's observations, I have included them in my analysis, as they are, so far as I can tell, equal in quality to those of Cook and King.

Two developments in late 20th-century technology have made a re-examination of Cook's longitude determinations possible. The first is the ready availability of personal computers. My own machine is not by any means state-of-the-art, yet it is easily possible for me to repeat all 122 calculations of longitude, including looking up refraction tables, correcting for parallax, and performing inverse interpolation on the Nautical Almanac positions of the Moon to determine Greenwich time, in less than two minutes. Originally this work must have taken at the very minimum ten days to complete, working diligently throughout the day, hour-after-hour, day-after-day.

The second development, is a means of accurately calculating the positions of objects in the solar system which is based, not on theoretical analysis of telescopic observations, but on direct measurements of their mass and movements using space probes, radar, and in the case of the Moon, laser signals bounced from quartz reflectors left behind by the Apollo astronauts. Possibly a good indication of the importance of the accuracy of the ephemerides used for this analysis is that, in calculating the positions of objects in the sky over Nootka two centuries ago, I have had to take into account the cumulative effect of fluctuations in the Earth's axial rotation due to tidal friction, even though these daily fluctuations are typically measured in fractions of a millisecond.

The way I tackled the analysis was to re-do Cook's calculations four ways. The first way, or analysis mode as I call it, was to take Bayly's figures as literally as possible, and re-compute the longitude using 18th-century tables and techniques. The second analysis mode, mode 2, was to take those mode 1 results where there was a discrepancy between Bayly's and the calculated longitudes and look at the possibility that there was a simple arithmetic or typographical error that could plausibly explain the difference. The objective of the mode 2 analysis was to reproduce Cook's results exactly. For mode 3, I corrected any errors that Cook had made and re-computed the longitudes, still however using 18th-century tables and techniques. And for mode 4, I re-worked all the calculations using modern tables and modern techniques.

For mode 1, only obvious errors were corrected—for example calculation shows that observations dated April 19 could only have been made the previous day. In seven cases for the Sun and six for the Moon the nature of the altitude was also incontrovertibly wrong. In fact the first surprise, and a foretaste of what was to come, came with the first look at the first result, which is an observation credited to King made on April 2, 1778 (Bayly 1782, 46). Actually this observation was made shortly before 4 pm in the afternoon on April 1 as Cook reckoned local time to be 16 hours ahead of Greenwich, not as we do today eight hours behind. The Sun's altitude is recorded as 62°35', a value which is only reached at noon on a mid-summer's day at Nootka. In fact, what had been recorded was the angle between the Sun's lower limb and the point immediately above the observer's head (the zenith); the actual altitude of the Sun corrected for refraction was only 27°39'.

Some of the results of these mode 1 computations agreed with Cook's results, and some did not. On the whole the comparison was not good. In only 63 of the 122 cases was the computed distance, cleared for parallax and refraction, within 10" of arc of the value that was used to compute the longitude given in Bayly's book (mean difference 8", stnd.dev. 15"). [Note: 1" (second of arc) = 1/3600 of 1° (degree)].

For the mode 2 analysis I looked for simple explanations for the mode 1 discrepancies. This was fairly successful. A typical example of what was found was the first observation using the star Regulus made by Cook on April 3. The given zenith distance of the Moon's upper limb (39°07') gives an altitude for the Moon which is too low by almost exactly its own diameter. Taking the table entry to be the zenith distance of the lower limb not only corrects the altitude by modern reckoning, but reduces the distance discrepancy from 13" to zero i.e. there is a “typo” in the table; ZD UL should be ZD LL. There are at least 18, and perhaps as many as 28 examples of this kind of error in Bayly's tables.

Four amusing calculation errors were discovered among the 31 observations credited to Bayly. In each case, whoever made the calculations failed to clear the lunar distances for refraction. This is a fairly major oversight, yet interestingly the uncleared distances give longitudes closer to Cook's mean longitude than they would have, had they been processed correctly.

The most difficult kind of error to deal with in the mode 2 analysis was the sextant index error. I spent a lot of time looking into these and am left with the distinct impression that the figures given by Bayly are not very reliable. The index error is the sextant reading when the two images of the same body are brought together. The reading ought to be zero, but in practice there is often a small variable off-set.

The difficulty with the hypothesis that some of the index errors are incorrectly tabulated is that by postulating a different index error one can always remove the distance discrepancy, and yet have no substantiating evidence that the proposed alternative error value is correct. However, in some cases, it was possible to accept that the errors had been carelessly recorded. For example, the two observations made by King on April 21 and 22 using the Dolland instrument appear to have been corrected by -15", even though Bayly's figures state he used -15" on the 21st and -30" on the 22nd. King used -13" for his observation with the third Ramsden instrument (R3) dated April 4, but Cook's result with R3 on the same day has been processed using +13". There are many other similar examples.

The end result of the mode 2 analysis was 103 cases out of 122 where the computed lunar distance agreed with Cook's to better than 10" of arc (mean difference 4", stnd.dev. 12").

Certainly more interesting than typographical errors which could have crept into Bayly's figures at any time after the original calculations were made, are those errors that were incorporated into the longitude calculations. There seem to be just as many of these errors as there were typographical errors. A typical example would be Bayly's second observation dated April 22. The given altitude for the Moon is by my reckoning exactly right, yet the 77°29' for the zenith distance of the Sun's upper limb puts the Sun 29', i.e. about one diameter, too low. Almost certainly what was measured was the lower limb, but the calculations show that the upper limb was nevertheless used in the longitude calculation.

To see the consequences of this type of error I re-calculated the mode 2 results, correcting any mistakes that had been found. This constituted the mode 3 analysis. And as indicated above, I then re-calculated the mode 3 results using my own version of the Nautical Almanac. This was based on positions of the Sun and Moon in 1778 generated by one of the Numerically Integrated Ephemerides developed at the California Institute of Technology's Jet Propulsion Laboratory at Pasadena.

The results of this work are shown in the Figures.

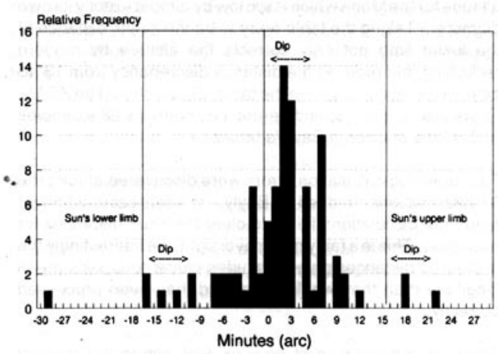

Figure 1: Sun Altitude Errors
Cook's - Modern values (excluding unknown dip)

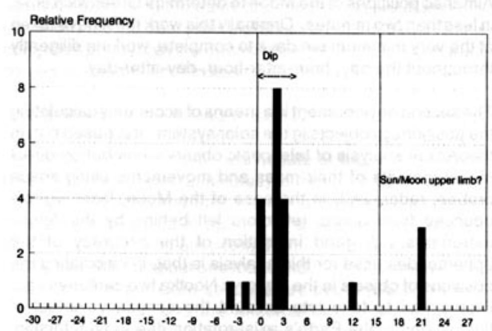

Figure 3: Regulus Altitude Errors
Cook's - Modern values (excluding unknown dip)

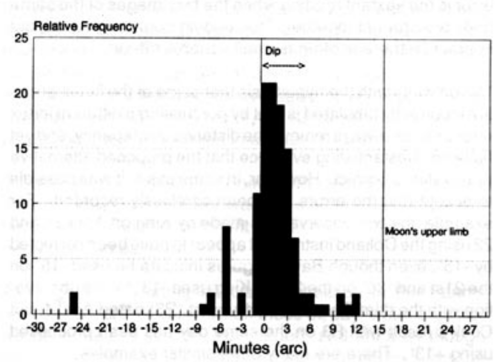

Figure 2: Moon Altitude Errors
Cook's - Modern values (excluding unknown dip)

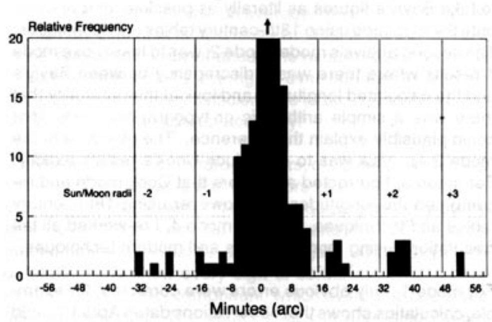

Figure 4: Anomalous Altitude Errors
Cook's - Modern values
Sun, Moon and Regulus

Figures 1 to 3 show histograms of the error in the altitude measurements used for the longitude calculations (mode 2). Altitudes of the Sun, Moon or star (Regulus) were only required for the parallax and refraction corrections of the lunar distance and very precise measurements were not necessary. Nevertheless, the 18th-century navigators usually made good observations as evidenced by their latitude determinations which were usually less than a minute of arc in error. Similarly precise measurements would have been used by Cook to establish the relationship between apparent (local solar) time and the time by his chronometers.

The standard deviation of the altitude errors in Cook's observations is about five minutes of arc, which is a bit high. One possible contributing factor, as indicated in the Figures, is that some unrecorded allowance was made for dip i.e. the height of the observer's eye above sea-level. If the altitudes in Bayly's tables need to be corrected for dip, a correction I did not make, then there would be a positive error in the range 3' to 6'. It seems unlikely however that such a routine correction would not be made: and it might not even have been necessary if due to the very restricted view of the open ocean at Ships' Cove, an artificial horizon [a basin of quicksilver (mercury)] was used.

There is a second possible explanation for the altitude errors; and that is that they are a result of timing errors. Ideally, the altitude and lunar distance measurements would have been made simultaneously, but it was recognized in the navigational instruction manuals of the time that a shortage of instruments and observers could easily preclude this. Instead, it was allowed that altitude measurements should be made as soon as possible, but less than a minute after the lunar distance had been fixed. The result of this procedure would have been that objects rising in the eastern sky would be recorded with altitudes too high, and objects setting in the western sky would be recorded with altitudes too low. The observations in Bayly's tables, particularly those of the Moon but also of the Sun, do in fact show such a bias. Regardless of whether objects were rising or setting, measurements of altitude appear to have been made too late. Moreover, the mean magnitude of the equivalent timing error of the Moon observations is 20 seconds, which is the delay that might be expected if the altitude was observed with the same instrument as was used to measure the lunar distance. If this explanation is correct, the positive bias in the altitude errors would then be attributable, not to dip, but to the preponderance of measurements (60%) made in the eastern hemisphere. Similarly the range of the errors would be attributable, not to indifferent measurement, but to varying time delays and rates of change of altitude of the bodies being observed.

All of the calculations that navigators make involving the position of the Sun and Moon are based on the position of the centre of the disc, but in practice observations of position are made relative to one or other of the edges (limbs) of the disc. To find the centre, the radius (semi-diameter) is then added or subtracted as required. Figure 4, which shows a composite histogram of the altitude errors, provides evidence that this routine operation was not always made with care. If for example, for an altitude measurement the upper limb is observed, but the radius is added instead of subtracted, the altitude will be out by two radii. If the correction is neglected, it will be out by one radius. And if, as Figure 4 suggests, the figures are re-worked without good records being kept, it is possible to make more than one such mistake and be out by three radii or more.

Two of Bayly's observations of the altitude of the star Regulus, which of course has no limbs, are out by the radius of the Sun, and in one case the incorrect altitude is not just a "typo", it was actually used! In one observation by Cook and one by King, the altitude of the star Regulus is out by the diameter of the Sun, though these incorrect values were not used.

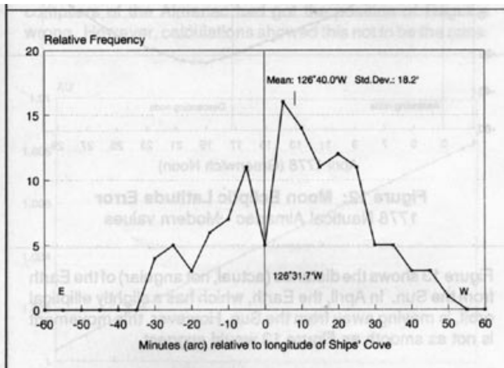

Figure 5: Cook, King and Bayly's Longitudes
As given in Bayly's Tables of Results

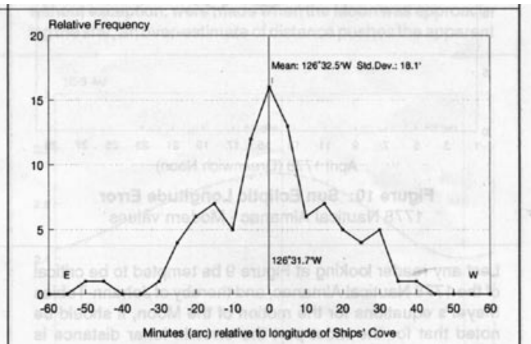

Figure 7: Cook, King and Bayly's Longitudes
Sun - Moon Results
1778 Nautical Almanac and other errors corrected

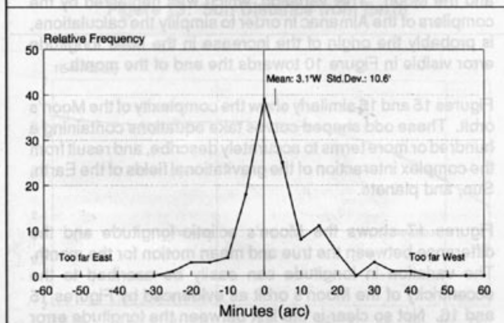

Figure 6: Corrections to Longitudes
Excluding Nautical Almanac corrections
Cook's result - computed result
(The main source of errors is improperly cleared altitudes)

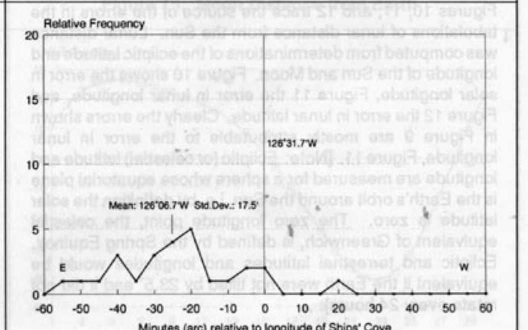

Figure 8: Cook, King and Bayly's Longitudes
Regulus - Moon Results
1778 Nautical Almanac and other errors corrected

So what was the effect of these and other errors on Cook's longitude determinations? The answer is not much. Figure 5 shows a histogram of longitude determinations as listed by

Bayly compared to the true longitude of Ships' Cove. The mean error is 8.3'W, slightly less than Cook's journal figure because I have included Bayly's observations. For observations using the Sun the error is 12.1'W, and using Regulus 5.7'E, a difference I shall come back to later. After correction (mode 3) the error becomes 5.3'W. For the Sun observations alone it becomes 10.3'W, and for the Regulus observations 13.3'E.

The non-Nautical Almanac errors evidently did contribute somewhat to Cook's westerly error, but as Figure 6 shows, on the whole these types of error tended to average out.

Figures 7 and 8 show the results after being re-worked using my modern version of the Nautical Almanac. Figure 7 shows the Sun observations; Figure 8 the Regulus observations. Figure 7 is a pleasing histogram; not only has the mean longitude error been virtually eliminated (0.8'W or 1.0 km), but the curve has a symmetry which suggests that individual observations were subject to a truly random error, as indeed they would be if the only remaining error were to be in the sextant readings of the lunar distance. Unfortunately Figure 8 shows that this is not true for the Regulus observations: the error here has actually increased to a surprising 25.0'E.

In order to check that the good results shown in Figure 7 were not a chance occurrence, I applied the same analysis technique to Vancouver and Whidbey's observations at Nootka in 1792. The result was equally good: their longitude can be corrected to within 0.5' (600 m) of the correct value.

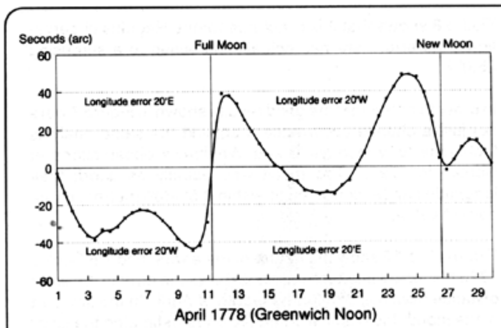

Figure 9: Sun - Moon Distance Error
1778 Nautical Almanac - Modern values

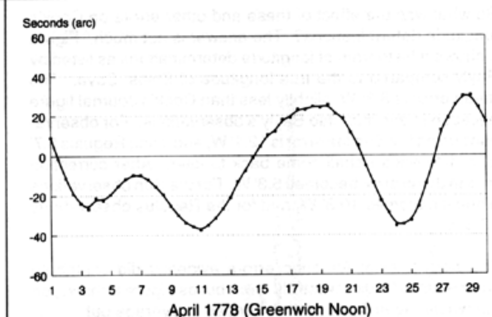

Figure 11: Moon Ecliptic Longitude Error
1778 Nautical Almanac - Modern values

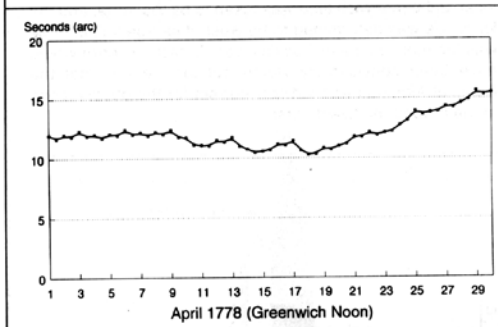

Figure 10: Sun Ecliptic Longitude Error
1778 Nautical Almanac - Modern values

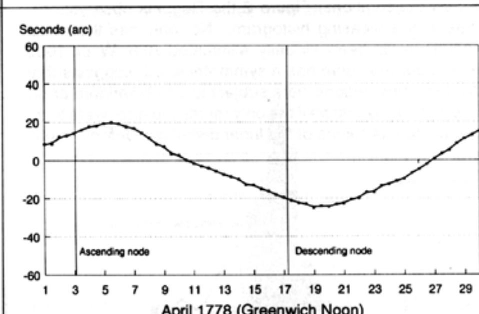

Figure 12: Moon Ecliptic Latitude Error
1778 Nautical Almanac - Modern values

Figures 9 to 12 show the nature of the errors made in Cook's 1778 Nautical Almanac.

Figure 9 shows the error in the lunar distance from the Sun for the month of April. In the first part of the month, the Nautical Almanac underestimated (negative error) the lunar distance. Since the Moon was moving away from the Sun, this led to estimates of Greenwich time which were too fast, and consequently the difference between local and Greenwich time was taken to be too large, and the longitude was determined to be west of its true value. This was the main cause of Cook's error. In fact, his error would have been considerably worse but for the fact that the Nautical Almanac was virtually correct for the observations made around the twenty-first.

Lest any reader looking at Figure 9 be tempted to be critical of the 1778 Nautical Almanac, and thereby of Johann-Tobias Mayer's equations for the motion of the Moon, it should be noted that for the most part the error in lunar distance is decidedly less than 1 minute of arc. This was the accepted goal of his time, and even today, non-professional astronomers would be hard pressed to do better.

Figures 10, 11, and 12 trace the source of the errors in the tabulations of lunar distance from the Sun. Lunar distance was computed from determinations of the ecliptic latitude and longitude of the Sun and Moon. Figure 10 shows the error in solar longitude, Figure 11 the error in lunar longitude, and Figure 12 the error in lunar latitude. Clearly the errors shown in Figure 9 are mostly attributable to the error in lunar longitude, Figure 11. [Note: Ecliptic (or celestial) latitude and longitude are measured for a sphere whose equatorial plane is the Earth's orbit around the Sun, i.e. by definition the solar latitude is zero. The zero longitude point, the celestial equivalent of Greenwich, is defined by the Spring Equinox. Ecliptic and terrestrial latitudes and longitudes would be equivalent if the Earth were not tilted by 23.5° and it did not rotate every 24 hours].

Some idea of the complexity of the Earth's and Moon's orbit in April 1778 can be seen in Figures 13 to 16.

Figure 13 shows the distance (actual, not angular) of the Earth from the Sun. In April, the Earth, which has a slightly elliptical orbit, is moving away from the Sun. However, this movement is not as smooth as Figure 13 would suggest.

Figure 14 clearly shows the variation in the Earth's velocity away from the Sun as it orbits the centre of gravity of the Earth and the Moon. This variation, which was neglected by the compilers of the Almanac in order to simplify the calculations is probably the origin of the increase in the solar longitude error visible in Figure 10 towards the end of the month.

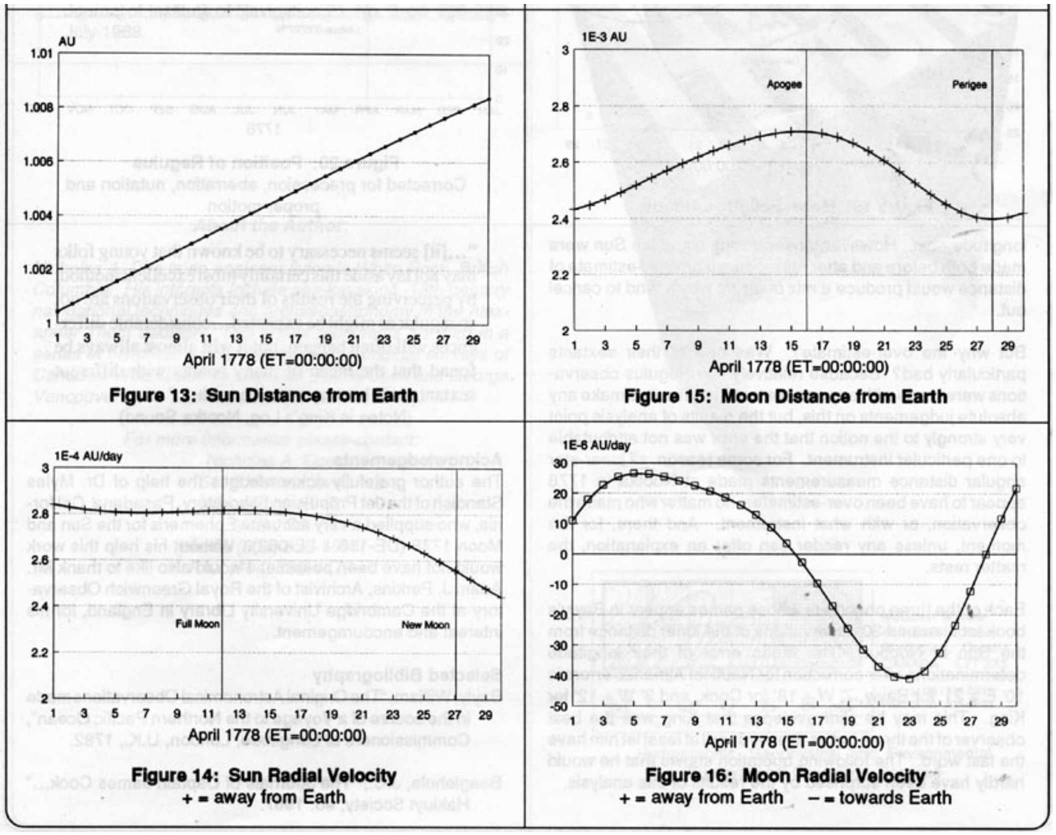

Figure 13: Sun Distance from Earth

Figure 15: Moon Distance from Earth

Figure 14: Sun Radial Velocity
+ = away from Earth

Figure 16: Moon Radial Velocity
+ = away from Earth — = towards Earth

Figures 15 and 16 similarly show the complexity of the Moon's orbit. These odd shaped curves take equations containing a hundred or more terms to accurately describe, and result from the complex interaction of the gravitational fields of the Earth, Sun, and planets.

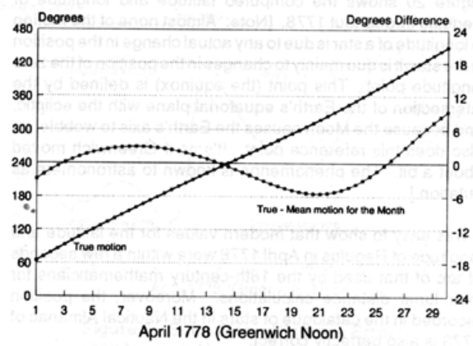

Figure 17: Moon Ecliptic Longitude

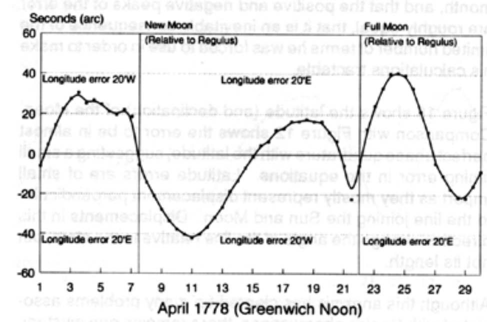

Figure 19: Regulus - Moon Distance Error
1778 Nautical Almanac - Modern values

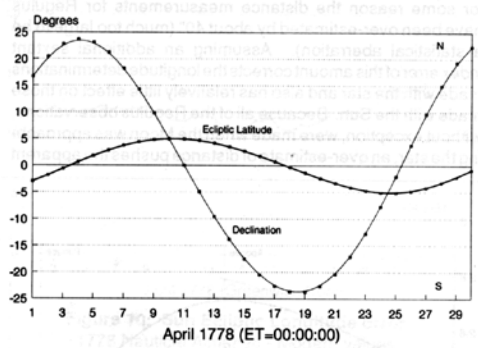

Figure 18: Moon Ecliptic Latitude

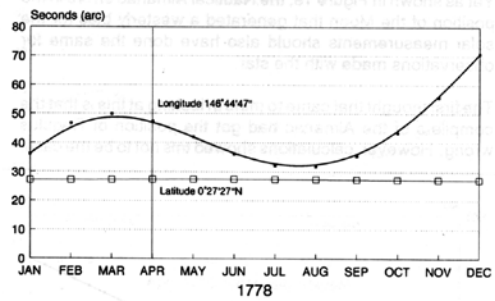

Figure 20: Position of Regulus
Corrected for precession, aberration, nutation and proper motion

Figures 17 shows the Moon's ecliptic longitude, and the difference between the true and mean motion for the month. The variation in longitude can easily be ascribed to the eccentricity of the Moon's orbit as evidenced by Figures 15 and 16. Not so clear is the link between the longitude error shown in Figure 11 and the actual longitude shown in Figure 17. Although I have not examined Mayer's equations for the Moon's motion in longitude, I suspect from the fact that the error appears to go through three complete cycles in the month, and that the positive and negative peaks of the error are roughly equal, that it is an inevitable consequence of the limited number of terms he was forced to use in order to make his calculations tractable.

Figure 18 shows the latitude (and declination) of the Moon. Comparison with Figure 12 shows the error to be in almost perfect phase quadrature with the latitude, suggesting a small timing error in the equations. Latitude errors are of small import as they mostly represent displacement perpendicular to the line joining the Sun and Moon. Displacements in this direction change the angle of the line relative to the stars, but not its length.

Although this analysis has cleared up many problems associated with Cook's observations, there remains one mystery. As shown in Figure 8, those determinations that rely on measurements of the distance between the Moon and the star Regulus give a corrected longitude which is 25.0' too far east. Yet as shown in Figure 19, the Nautical Almanac errors in the position of the Moon that generated a westerly bias for the solar measurements should also have done the same for observations made with the star.

The first thought that came to mind in looking at this is that the compilers of the Almanac had got the position of Regulus wrong. However calculations showed this not to be the case. Figure 20 shows the computed latitude and longitude of Regulus throughout 1778. [Note: Almost none of the motion in longitude of a star is due to any actual change in the position in the star; it is due mainly to changes in the position of the zero longitude point. This point (the equinox) is defined by the intersection of the Earth's equatorial plane with the ecliptic, and because the Moon causes the Earth's axis to wobble, so also does this reference point. It's as if Greenwich moved about a bit. The phenomenon is known to astronomers as nutation].

It was easy to show that modern values for the latitude and longitude of Regulus in April 1778 were within a few seconds of arc of that used by the 18th-century mathematicians for their lunar distance calculations. Moreover, the position recorded in the catalogue of stars in the Nautical Almanac of 1773 is also perfectly correct.

The only other explanation I have for the discrepancy is that for some reason, the distance measurements for Regulus have, on average, been over-estimated by about 42". Assuming an additional sextant index error of this amount corrects the longitude determinations made with the star. It also has relatively little effect on the mean value of those made with the Sun, although it does significantly increase their standard deviation. Because all of the Regulus observations, without exception, were made when the Moon was approaching the star, an over-estimate of distance pushes the apparent longitude east. However, observations using the Sun were made both before and after full moon, so an over-estimate of distance would produce a mix of errors which tend to cancel out.

But why the over-estimate? Was one of their sextants particularly bad? Because relatively few Regulus observations were made with any one sextant it is difficult to make any absolute judgements on this, but the results of analysis point very strongly to the notion that the error was not attributable to one particular instrument. For some reason, all lunar-star angular distance measurements made at Nootka in 1778 appear to have been over-estimated compared to the lunar-Sun measurements, no matter who made the observation, or with what instrument. And there, for the moment, unless any reader can offer an explanation, the matter rests.

Each of the three observers whose names appear in Bayly's book took around 30 observations of the lunar distance from the Sun at Nootka. The mean error of their longitude determinations after correction for Nautical Almanac errors is 10'E $\pm$21' for

Bayly, 7'W ±18' for Cook, and 3'W ±12' for King. This may be slim evidence that King was the best observer of the three: but as a reward, I will at least let him have the last word. A quotation that shows that he would hardly have been surprised by the results of this analysis.

...(it) *seems necessary to be known that young folks may not lay aside this certainly most excellent method, by perceiving the results of their observations are not so regular as might be expected ... considerable differences will often happen, but it will almost allways be found that the mean of many results with different sextants will be very near the truth....*
Notes in King's log, Nootka Sound

* * * * * * * * * *

The author is an engineer living in White Rock. His interests include sea-kayaking, 18th-century navigational techniques, and archaeoastronomy in the Alexander Thom tradition.

ACKNOWLEDGEMENTS

The author gratefully acknowledges the help of Dr. Myles Standish of the Jet Propulsion Laboratory, Pasadena who supplied a very accurate Ephemeris for the Sun and Moon 1778 (DE-118 + LE-062). Without his help, this work would not have been possible. I would also like to thank Mr. Adam J.Perkins, Archivist of the Royal Greenwich Observatory at the Cambridge University Library in England, for his interest and encouragement.

SELECTED BIBLIOGRAPHY

Bayly, William. 1782. *The Original Astronomical Observations made in the course of a Voyage to the Northern Pacific Ocean*. London: Commissioners of Longitude.

Beaglehole, J.C., ed. 1967. *The Journals of Captain James Cook....* Hakluyt Society.

Commissioners of Longitude. 1778. *The Nautical Almanac and Astronomical Ephemeris*. London.

Doe, N.A. 1992. *Where was Nootka in 1792? - An Explanation of Captain Vancouver's Longitude Error.* Unpublished.

H.M. Nautical Almanac Office. April 1966. *A Modern View of Lunar Distances*. Journal of Institute of Navigation 19 No.2:131-153.

Maskelyne, Nevil. Astronomer Royal. 1776. *Tables Requisite to be Used with the Astronomical and Nautical Ephemeris*. London.

Newhall,X.X., Standish,E.M. and Williams,J.G. 1983. *DE-102: A Numerically Integrated Ephemeris of the Moon and Planets....* Astronomy and Astrophysics 125:150-167.

Petze, Charles L. 1948. *The Evolution of Celestial Navigation.* New York: Motor Boating, Hearst Magazines Ideal Series Vol.26.

Sadler,D.H. 1968. *Man is not Lost... The Nautical Almanac 1767-1967*. London: HMSO.

Taylor,E.G.R. July 1968. *Navigation in the Days of Captain Cook*. Journal of Institute of Navigation 21 No.3:256-276.