The Potential Accuracy of the Eighteenth-Century Method of Determining Longitude at Sea

by Nicholas A. Doe

If one ignores, as we in the western world are wont to do, the achievements of the Polynesians who were sailing the Pacific Ocean more than a thousand years before Cook, it can be said that it was not until the 18th century that position determinations at sea were made with any accuracy. That century saw the development of both the sextant and the chronometer—the essential tools of the navigator right up until the advent of the electronics-based systems of today.

The double-reflecting mirror principle used in a sextant[1] was first enunciated by Sir Isaac Newton in 1699, and the first practical instruments were made independently in 1730 by the Englishman, John Hadley and the American, Thomas Godfrey. By the latter half of the 18th century, the skill of sextant makers like Jesse Ramsden, Peter Dollond, John Bird, George Adams and others had made it possible for navigators at sea to determine their latitude to within one or two nautical miles.[2] The determination of longitude with the same accuracy however remained an elusive goal. The 18th-century quest was a method that required only a sextant,[3] for although John Harrison's chronometers demonstrated accuracies of better than 30 nautical miles over several months—the goal set by the British Longitude Act of 1714, there remained in official circles the suspicion that chronometers were expensive and unreliable contrivances. The lunar-distance method based on the lunar tables of the astronomer Johann Tobias Mayer came closest to their ideal, and with the support of Nevil Maskelyne, Astronomer Royal from 1765 to 1811, the method of lunar-distances, or lunars as it is known, remained the method of choice, particularly of the British navy, until accurate, mass-produced chronometers came into widespread use in the early-19th century.[4]

Because the Earth rotates about its axis at a near-constant rate of fifteen degrees per hour, the determination of longitude can be reduced to the task of determining the difference between local time, which is often established at noon, and the time on the Greenwich meridian. Trying to establish Greenwich time by measuring the position of the Moon in its monthly orbit relative to the Sun or stars was a bit like using a watch with an hour hand that goes around once every 29 days instead of twice every 24 hours. To measure longitude to within, say, two nautical miles (3.0' at a latitude of 49°, 12 seconds of time), the angle between the Moon and the Sun or a star (the lunar distance) had to be established to about six seconds of arc. This is an impossible task for even the best of 20th-century instruments when uncertainties introduced by weather-dependent refraction and tricks on the eye played by bright light are included.

The 18th-century navigators however were undaunted.[5] They were accustomed to working with inaccurate measuring instruments and were masters of the art of minimizing the effects of suspected errors by taking readings in circumstances designed to ensure that such errors were equal and opposite, and thereby nullified. Captain James Cook, and the astronomers who accompanied him on his voyages, often made several hundred observations of the Moon’s position using several different instruments to determine their longitude, and Cook’s one-time midshipman Captain George Vancouver was later on one occasion to make over two thousand such observations in an attempt to accurately fix his position.[6]

Yet in spite of this, the accuracy of the longitude determinations made by Cook and Vancouver and others was less than might have been expected. It must have been particularly galling, for example, for Vancouver to find in October 1792 that his reckoning of the longitude of Nootka, British Columbia was 20.5' (13.3 nautical miles) east of that made by Cook, Lieutenant King, and the astronomer William Bayly in April 1778, even though both expeditions had made over six hundred individual measurements of the Moon’s position.[7]

The sources of error in the method of lunars were numerous and included sextant calibration; refraction; parallax, which is especially a problem with observations of the Moon because it is so close to us; the varying sizes of the Sun and Moon’s discs which unfortunately for navigators do not come with a black dot marking their centres; arithmetic mistakes, which were inevitable given the complexity of the calculations and the absence of calculators; and the lunar-distance tables in the Nautical Almanac.[8] Modern lunar theory is sufficiently precise for us to be able to go back and correct the errors in the tables in the 18th-century editions of the Nautical Almanac and these corrections clearly point to the almanac errors as being a serious source of the inaccuracies. In an experiment to determine the potential accuracy of the method of lunars, I have made a statistical analysis of some longitude determinations made by Cook and Vancouver after correcting for the errors that their almanacs contained.

The approach I took for the analysis was as follows. Step 1: set up, on a computer, a random number generator which can generate “errors” with a zero mean and a normal (Gaussian) distribution and a controllable standard deviation. Round off these numbers to the nearest minute of arc to correspond with the likely (useful) resolution of the instruments and let this random number generator simulate the unknown errors in individual lunar-distance measurements. Step 2: Take six numbers generated at step 1 and average them. This simulates the common practice of the navigators of making “sets” of measurements comprising averaged distances, averaged altitudes, and averaged watch-times. Step 3: Take the same number of sets per group of sets as did the navigators in their reports and average the result. Vancouver, for example, reports the results of 16 groups of sets ranging in size from 2 to 8 sets in his Nootka longitude determination. Step 4: For each group of sets work out from the average lunar-distance error the equivalent longitude error.[9] Step 5: Compute the mean and standard deviation of the longitude errors calculated at step 4. Step 6: Adjust the standard deviation of the simulated instrument readings generated at step 1 until the statistics of the simulated longitude errors at step 5 match those of the original observations, corrected for errors in the Nautical Almanac. Vancouver’s results at Nootka, for example, have a standard deviation of eleven minutes of arc of longitude after correction.[10]

The result of this process applied to Cook’s results at Nootka,[11] Vancouver’s at Nootka,[10] and Vancouver’s at Port Discovery in May 1792[12] was an estimated standard deviation of the error in individual sextant measurements of lunar-distance of respectively 1.7', 2.3', and 1.9'. These results are pleasingly consistent, and within the expected range of accuracy of 18th-century sextants when it is borne in mind that the errors include errors made in calibration, and arithmetic and other mistakes made while correcting for refraction and parallax. It is of course possible that the assumption made at steps 1 and 2 that the errors within a set were statistically independent is at least partially wrong. Measurements were usually made in rapid succession and with the same instrument; however, even supposing that the same error applied to all of the measurements within a set, which is as “un-independent” as they could be, then it would only be necessary to reduce the above estimates by $\sqrt{6}$ to give standard deviations of 0.7', 0.9', and 0.8'. This is still a plausible result.

Assuming that the actual standard deviation of a sextant measurement was about 2', and that the error had a zero mean, then a longitude determination made on the basis of say 100 sets of 6 measurements using a perfectly accurate Nautical Almanac would have an expected accuracy of $2' \times 29.6/\sqrt{600}$ = 2.4' (1.6 nautical miles at a latitude of 49°). That this degree of accuracy was in principle achieved is evidenced by the results of recalculation of some of the determinations reported in earlier papers. If one ignores the problematic observations of the distance between the Moon and the star Regulus, then Cook's corrected longitude error at Nootka based on 576 measurements (96 sets) was only 0.8' (W). Vancouver's error at Nootka based on 600 measurements (100 sets) was even less at 0.5' (W). Vancouver's error at Port Discovery based on 1320 measurements (220 sets) is estimated to be 4.3' (E), but this could well be too high because the observations have been wrongly dated. These results show how close the navigators actually were to their ambitious goal of measuring their longitudes as accurately as they did their latitudes.

Unfortunately, making hundreds of measurements of the Moon's position on a regular basis was totally impractical, [13] and hence the eventual demise of the method of lunars in favour of chronometers. This should not however detract from an appreciation of the skill of the instrument makers or of those that used them. That the desired accuracy was not achieved was a failing of astronomical theory rather than the instrument-makers' or observers' practice.

Notes and References

1. A sextant is a 60º instrument capable of measuring angles by double reflection up to 120º. Early forms of the instrument were actually octants (45º instruments) but these were widely known by navigators as quadrants because they measured up to 90º. Throughout this paper the term "sextant" should be taken as including octants.

2. Stimson, A., "The Influence of the Royal Observatory at Greenwich upon the Design of 17th and 18th Century Angle-Measuring Instruments at Sea", Vistas in Astronomy 20, pp.123–130, 1976.

3. Sadler, D.H., "Man is not lost", HM Stationery Office, London, 1967.

4. Taylor, E.G.R., "The Haven-Finding Art - A History of Navigation from Odysseus to Captain Cook", Hollis & Carter, London, 1971.

5. William Wales, the astronomer on Cook's second voyage, once opined that "with very little trouble" the method of lunars could be used to fix longitude to within 10' of arc. (Wales and Bayly, "Astronomical Observations... ", pp. xlvii–xlix, 1788). The implied accuracy of the Mayer-Mason lunar tables is about 20" which was flattering. Mayer originally had only claimed 2', but after improvements in 1777 and again in 1789 this estimate was reduced to 1' and then 30". In fact, errors in the 30" to 1' range were common. I thank Andrew David for the reference.

6. 346 sets of 6 at Observatory Inlet, British Columbia, August 1793. Lamb, W.Kaye, ed., "A Voyage of Discovery ... by George Vancouver", Hakluyt Society, pp.1026–1028, 1984.

7. Lamb, "A Voyage...", p.685.

8. Sadler, D.H., "Lunar Distances and the Nautical Almanac", Vistas in Astronomy 20, pp.113–121, 1976.

9. On average, 1' of lunar-solar distance equates to 30.1' of longitude, the duration of a synodic month in days plus an allowance for the fact that the distance does not usually vary over the full 180º, i.e., we do not have eclipses every month. Observations in the 18th century were only made in the distance range 40–110º which reduces the equivalence , on average, to 29.6' of longitude when the Sun is used as a reference, and roughly 27.3' of longitude when a suitably-positioned star is used as a reference. The relationship at any given moment depends on the orbital velocity of the Moon which varies considerably.

10. Doe, N.A., "Where was Nootka in 1792? An Explanation of Captain Vancouver's Longitude Error", Lighthouse 47, pp.15–18, Spring 1993.

11. Doe, N.A., "An Analysis of Captain Cook's Longitude Determinations at Nootka, April 1778", Lighthouse, 48, pp.21–29, Fall 1993.

12. Doe, N.A., "Captain Vancouver's Longitudes—1792", Journal of Navigation, Vol.48, pp.374–388, September 1995.

13. It was estimated that the mathematical processing of just one set of lunar distance observations took four hours. This was later improved by the use of tables, but even so, it is unlikely that the time was reduced to below 30 minutes even for the most practised of navigators.